

## Surfaces of revolution with constant Gauß curvature

A surface of revolution is usually described by giving its meridian curve  $s \mapsto (r(s), h(s))$ . The surface is then obtained by rotation:

$$(x, y, z) := (r \cos \varphi, r \sin \varphi, h).$$

Any kind of curvature condition can be expressed as a differential equation for the meridian curve.

The case of constant Gauß curvature  $K$  is particularly simple if the meridian is parametrized by arclength, i.e.,  $r'^2 + h'^2 = 1$ . In this case the meridian is determined by

$$r''(s) + K \cdot r(s) = 0, \quad h(s) = \int_0^s \sqrt{1 - r'(t)^2} dt.$$

We describe the three kinds of examples in the case  $K = 1$ .

Sphere:

$$r(s) = \sin(s), \quad 0 \leq s \leq \pi$$

With cone points:

$$r(s) = a \sin(s), \quad 0 \leq s \leq \pi, \quad 0 < a < 1$$

With singularity curve:

$$r(s) = a \sin(s), \quad b \leq s \leq \pi - b, \quad 1 < a, \quad \cos(b) := 1/a.$$

In the program 3DXM the user can change the parameter  $a$  in the Settings Menu,  $a = cc$ . See also the default morph.