

Möbius Strip and Klein Bottle *

Other non-orientable surfaces in 3DXM:
Cross-Cap, Steiner Surface, Boy Surface

The **Möbius Strip** is the simplest of the non-orientable surfaces. On all others one can find Möbius Strips. In 3DXM we show a family with ff halftwists (non-orientable for odd ff , $ff = 1$ the standard strip). All of them are *ruled surfaces*, their lines rotate around a central circle. Möbius Strip *Parametrization*:

$$F_{\text{Möbius}}(u, v) = \begin{pmatrix} aa(\cos(v) + u \cos(ff \cdot v/2) \cos(v)) \\ aa(\sin(v) + u \cos(ff \cdot v/2) \sin(v)) \\ aa u \sin(ff \cdot v/2) \end{pmatrix}.$$

Try from the View Menu: **Distinguish Sides By Color**. You will see that the sides are not distinguished—because there is only one: follow the band around.

We construct a **Klein Bottle** by curving the rulings of the Möbius Strip into figure eight curves, see the Klein Bottle *Parametrization* below and its **Range Morph** in 3DXM.

$$w = ff \cdot v/2$$

$$F_{\text{Klein}}(u, v) = \begin{pmatrix} (aa + \cos w \sin u - \sin w \sin 2u) \cos v \\ (aa + \cos w \sin u - \sin w \sin 2u) \sin v \\ \sin w \sin u + \cos w \sin 2u \end{pmatrix}.$$

* This file is from the 3D-XplorMath project. Please see:

<http://3D-XplorMath.org/>

Actually, there are *three different Klein Bottles* which one cannot deform into each other through immersions. The best known one has a reflectional symmetry and looks like a weird bottle. The other two are mirror images of each other. Along the central circle one of them is left-rotating the other right-rotating. See the **Default Morph** of the *Möbius Strip* or of the *Klein Bottle*: both morphs connect a left-rotating to a right-rotating surface.

On the *Boy Surface* one can see different Möbius Strips. The **Default Morph** begins with an equator band which is a Möbius Strip with *three halftwists*. As the strip widens during the deformation it first passes through the triple intersection point and at the end closes the surfaces with a disk around the center of the polar coordinates.

Moreover, each meridian is the centerline of an *ordinary Möbius Strip*. Our second morph, the **Range Morph**, rotates a meridian band around the polar center and covers the surface with embedded Möbius Strips. - We suggest to also view these morphs using **Distinguish Sides By Color** from the View Menu.

On the *Steiner Surface* and the *Cross-Cap* the Möbius Strips have self-intersections and are therefore more difficult to see. The **Default Morph** for the *Steiner Surface* emphasizes this unusual Möbius Strip. - The **Range Morph** of the *Cross-Cap* shows a family of embedded disks, except at the last moment, when opposite points of the boundary are identified, covering the self-intersection segment twice.