The Helicoid - Catenoid Family*

Lagrange found in 1762 the minimal surface equation as Euler-Lagrange equation for the area minimization problem. Geometrically, this equation means that the mean curvature is 0. He found no other solution than the plane. In 1776 Meusnier discovered that the Helicoid and the Catenoid are also solutions. The next solutions were found 60 years later by Scherk.

The **Helicoid** $(aa = 0)$ − **Catenoid** $(aa = 1)$ **Family:**

$$\begin{aligned}
x(u, v) :=\, & \cos(aa \cdot \pi/2) \sin(u) \sinh(v) + \\
& \sin(aa \cdot \pi/2) \cos(u) \cosh(v)) \\
y(u, v) :=\, & \cos(aa \cdot \pi/2) \cos(u) \sinh(v) + \\
& \sin(aa \cdot \pi/2) \sin(u) \cosh(v) \\
z(u, v) :=\, & \cos(aa \cdot \pi/2) \cdot u + \sin(aa \cdot \pi/2) \cdot v
\end{aligned}$$

These formulas describe, for each aa, a minimal surface. Moreover, these surfaces are all isometric to each other; their Riemannian metrics, in the given parametrization, all have the same expression. The parameter lines $u = const$ on the Helicoid $(aa = 0)$ are straight lines and no other minimal surface is such a *ruled* surface. The Catenoid $(aa = 1)$ is the only minimal surface of revolution. These two are the only embedded surfaces in the aa-family.
The isometric aa-deformation is very fascinating, use `Cyclic`

* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

`Morph` in the Animation Menu. The small parameter squares do not change size or shape during the deformation, they just get bent a little. If the two sides of the surfaces are colored differently and aa varies from $aa = 1$ to $aa = 3$, one can observe that the Catenoid is turned inside out. The deformation period is 4 and really amazing to watch.

H.K.