The Waves Category*

The subject of wave phenomena is an exceedingly rich and varied one, playing an important rôle in everyday life, in the pure and applied sciences, and in many areas of mathematics. Water waves can be small ripples generated by a passing breeze or enormous, destructive tsunamis generated by earthquakes, which themselves are waves that travel in the Earth's crust. Maxwell's Equations of electromagnetism are wave equations that describe the behavior of light, and of the signals responsible for radio, TV, and the World Wide Web. The probability waves of quantum theory govern the behavior of elementary particles at the smallest spatial scales, and the search for gravitational waves at the largest scales is a major concern of modern cosmology.

Our goal in the 3D-XplorMath Wave Category is to show examples of waves from a fairly limited region of this vast landscape. Namely, the program displays the evolution in time of a one-dimensional "wave-form", by which we mean a real or complex valued function $u(x, T)$ of the "time", T, and a single "space" variable x. At a time T, the wave has a given "shape", the graph of the function $x \mapsto u(x, T)$, and the program displays this graph with x as abscissa and $u(x, T)$ as ordinate.

[NOTE: The resolution of this graph is tResolution, and its

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* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

domain runs from tMin to tMax. That is to say, the interval [tMin,tMax] is divided in tResolution points $x_i, i = 1, \ldots,$ tResolution with spacing xStep $= \frac{(\text{tMax}-\text{tMin})}{(\text{tResolution}-1)}$, and the graph is plotted at the points (x_i, y_i) where $x_i =$ tMin$+(i-1)$ xStep and $y_i = u(x_i, T)$, and then these plot points are joined into a polygonal graph. Note that the t of tMax tMin and tResolution have nothing to do with the time T! Thus to make the graph smoother, you must increase tResolution in the Set Resolution & Scale... dialog, and to change the domain of the graph, you must change tMin and tMax in the Set t,u,v Ranges... dialog.]

The time evolution of the wave-form is shown by the standard flip-book animation technique; i.e., we plot the above graph for

$$T = \text{InitialTime} + k{\cdot}\text{StepSize},\ k = 0, 1, \ldots,$$

until the user clicks the mouse. The variables InitialTime and StepSize are set in the dialog brought up by choosing ODE Settings... from the Settings menu. To make the animation slower (but smoother) decrease StepSize, and conversely increase StepSize to make the wave-form evolution proceed more rapidly, but more jerkily.

As usual, formulas defining a wave form $u(x, T)$ can depend on the parameters aa,bb, ..., ii as well as on x and T, and for each of the canned wave-forms these formulas can be checked by choosing About This Object... from the Waves main menu.

In trying to understand features of a wave-form, it helps to see not only the animated evolution as above, but also to display the graph of the function $u(x, T)$ as a surface, over the (x, T)-plane or to show "time-slices" of this graph. Therefore the Waves menu has choices to permit the user to switch between these display methods. (These two alternate formats, being three dimensional objects, can be viewed in stereo.)

There is also a User Wave Form... item in the Wave menu, which brings up a dialog permitting the user to enter a formula for $u(x, t)$ as a function of x, t,aa, $\ldots$,ii.

Interesting wave-forms generally arise as solutions of so-called "wave equations". These are partial differential equations of evolution type for a function $u(x, t)$. That is, the function $u(x, t)$ is determined as the unique solution of a PDE satisfying some initial conditions. Perhaps the simplest example is the so-called "linear advection equation" $u_t + vu_x = 0$ (where v is some constant "velocity"). This has the general solution $u(x, t) = f(x - vt)$. But there are also many interesting non-linear wave equations, in particular the so-called soliton equations, including the Korteweg-DeVries (KDV), Sine-Gordon, and Cubic Schroedinger equations. Many of our examples are pure soliton solutions of these latter examples.

For a fairly detailed account of the elementary theory of wave equations, select *Introduction to Wave Equations* from the Topics submenu of the Documentation menu.

R.S.P.