Two Planar and Two Catenoid Ends

This Minimal surface is parametrized by a sphere with four punctures. In the complex plane its Weierstrass data are

$$\mathrm{gaus}(z) := \frac{z-a}{a\,z+1}\cdot z^2, \quad dh := \frac{z-1/z-a+1/a}{(z-1/z-c+1/c)^2}\cdot\frac{dz}{z}.$$

The Gauss map has a simple zero and pole at a and $-1/a$. These are finite points on the minimal surface since they are cancelled by the simple zeros of dh at $a, -1/a$. And the Gauss map has a double zero and pole at $0, \infty$. These give planar ends because dh is neither 0 nor ∞ there. The double poles of dh at $c, -1/c$ give catenoid ends because the Gauss map has simple finite values there - see `About Minimal Surfaces` in the Documentation Menu.

Catenoid ends usually come with a period. A residue computation shows that the choice $a := c - 1/c$ makes the period of the real part of the Weierstrass integral zero.

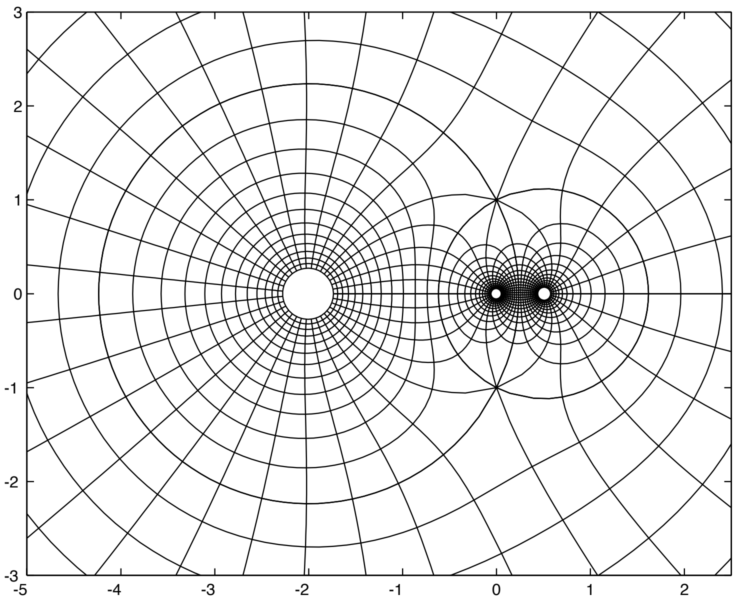
Note that the Weierstrass data are symmetric with respect to 180° rotation around i, $-i$. This corresponds to a 180° rotation of the minimal surface around the y-axis in $\mathbb{R}^3$.

Recall that the *Lopez-Ros theorem* states that the only complete *embedded* minimal surfaces that are parametrized by punctured spheres (and have finite total curvature) are *the plane and the catenoid.*

One can view the shape of the current surface as made from two copies of our Lopez-Ros-No-Go example by stacking these above each other and deforming the two half catenoids between them into the handle of the current surface between its two planes. The new surface is therefore another failing attempt of a counter example to the Lopez-Ros theorem. View in the Animate Menu the morph: `Watch The closed Catenoids Tilt`. It shows: The two catenoid ends get more vertical as c approaches 0, but in the limit the whole surface degenerates to a doubly covered plane.

Compare the current surface also to the *Riemann minimal surface*. Its shape can be viewed as an infinite stack of copies of the current surface – again, of course, with the half-catenoids between adjacent copies deformed into handles which join the copies.

Minimal surfaces look much better when the grid lines in the parameter domain behave like polar centers around the punctures. With this in mind, the current surface was parametrize by the following grid in $\mathbb{C}$:



H.K.