

Parabola

See also: Ellipse, Hyperbola, Conic Section and their ATOs, and in the Category Surfaces see: Conic Sections and Dandelin Spheres

The parametric equations for the Parabola are

$$x(t) := t^2/4p$$

$$y(t) := t,$$

where $p = aa/4$,

so the Parabola visualizes the graphs of the two functions $y(x) := \sqrt{4p \cdot x}$ and $x(y) := y^2/4p$.

The vertical line $x = -p$ is called the directrix and the point $(x, y) = (p, 0)$ is called focal point of the Parabola. The distance from a point $(x, y = \sqrt{4p \cdot x})$ on the Parabola to the directrix is $(x + p)$, and this is the same distance as from $(x, y = \sqrt{4p \cdot x})$ to the focal point $(p, 0)$, because $(x - p)^2 + y^2 = (x + p)^2$.

The point $(p, 0)$ is called "focal point", because light rays which come in parallel to the x-axis are reflected off the Parabola so that they continue to the focal point. This fact is illustrated in the program. It gives the following ruler construction of the Parabola:

Prepare the construction by drawing x-axis, y-axis, directrix and focal point F. Then draw any line parallel to the x-axis and intersect it with the directrix in a point S. The line orthogonal to the connection SF and through its midpoint is the tangent of the Parabola and intersects therefore the incoming ray in the point of the Parabola which we wanted to find.

The same construction works for Ellipse and Hyperbola, if the directrix is replaced by a circle of radius $2 \cdot a$ around one focal point. The curve is the set of points which have the same distance from this circle and the other focal point.

The Action Menu of the Parabola has an entry “Show Normals Through Mouse Point”. This illustrates an unexpected property of the Parabola. One may already be surprised that at the intersection points of normals always **three** normals meet. We know no other curve which is accompanied by such a net of normals. The surprise should increase if one looks at the y -coordinates of the parabola points from where three such intersecting normals originate: these y -coordinates add up to 0! In other words, the inter-

section behaviour of the normals reflects the addition on the y -axis.

The explanation of where this intersection property comes from is quite interesting. The normals of the Parabola are the tangents to its evolute, the semi-cubical parabola, a singular cubic curve (see Cuspidal Cubic). So the intersection property of the parabola normals can be thought of as defining an addition law for the evolute, and as such it is a simpler limiting case of the addition law that exists on any cubic curve.