

About the Schoen No-Go Theorem

H. Karcher

R. Schoen's characterization of the Catenoid says:

The only finite total curvature complete, embedded minimal surface having **two** ends, is the Catenoid.

This example shows what happens if one tries—in spite of this theorem of Schoen—to add a handle to a Catenoid.

The fundamental piece is similar to that of the catenoid fence, except that the handle does not go outward to the neighbouring catenoid but goes inward to meet its other half. However a gap remains, and as one tries to close it (by morphing with the modulus, aa , of the underlying rectangular torus) the surface degenerates to look almost like two catenoids that move farther apart as one tries to close the gap. We show this with the default morphing. The deformation goes between rather extreme surfaces where one has to adjust how far one computes into the end and then also

the size. While this animation is a bit jumpy, it is instructive and therefore recommended.

For a discussion of techniques for creating minimal surfaces with various qualitative features by appropriate choices of Weierstrass data, see either [KWH], or pages 192–217 of [DHKW].

[KWH] H. Karcher, F. Wei, and D. Hoffman, The genus one helicoid, and the minimal surfaces that led to its discovery, in “Global Analysis in Modern Mathematics, A Symposium in Honor of Richard Palais’ Sixtieth Birthday”, K. Uhlenbeck Editor, Publish or Perish Press, 1993

[DHKW] U. Dierkes, S. Hildebrand, A. Kuster, and O. Wohlrab, Minimal Surfaces I, Grundlehren der math. Wiss. v. 295 Springer-Verlag, 1991