

Nephroid of Freeth*

This curve, first described 1879, is the member $aa = 0$ in the following family of curves:

$$\begin{aligned}x(t) &= (1 - aa \cdot \sin(t/2)) \cos(t) \\y(t) &= (1 - aa \cdot \sin(t/2)) \sin(t)\end{aligned}$$

The default morph starts at $aa = 0$ with a circle, traversed twice. For small $aa > 0$ one double point develops. At $aa = 1$ the curve reaches the origin with a cusp. This cusp deforms into a second double point. At $aa = \sqrt{2}$ the two tangents of the double point coincide and are vertical. This point of double tangency deforms into three double points. The Nephroid of Freeth is reached at $aa = 2$, when two of the mentioned three double points coincide with the earliest one to form a *triple intersection*.

Apart from being in a simple family, which shows all these singularities of curves, we learnt from

www.2dcurves.com/derived/strophoid.html

that the Nephroid of Freeth has the curious property that one can construct a regular sevensgon with it: The vertical tangent at the triple intersection meets the curve again in two points whose radius vectors enclose the angle $3\pi/7$.

* This file is from the 3D-XplorMath project. Please see:

<http://3D-XplorMath.org/>