

## Hyperbolic Tangent\*

Definition:

$$\begin{aligned}\tanh(z) &:= \frac{\exp(z) - \exp(-z)}{\exp(z) + \exp(-z)} \\ &= \frac{w - 1}{w + 1} \text{ with } w = \exp(z)^2\end{aligned}$$

$$\tan(z) := -i \cdot \tanh(iz) = \frac{\sin(z)}{\cos(z)}$$

In 3DXM:  $\tanh(aa \cdot z/2) + bb \cdot z/2$ ,  $bb = 1 - aa$

default:  $aa = 1$ ,  $bb = 0$ , morph:  $0 \leq aa \leq 1$ .

The default visualization of this function is on the standard Cartesian Grid. The exponential map transforms this grid into a polar grid. The rational map  $w \mapsto (w - 1)/(w + 1)$  sends the polar centers at  $0, \infty$  to polar centers at  $-1, 1$ . The default morph connects  $z/2 \mapsto \tanh(z/2)$  to the identity.

Recall that all real functions that have power series representations can be extended to be functions over part of the complex plane. Of course this includes all functions that have simple definitions in terms of the exponential map.

H.K.

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\* This file is from the 3D-XplorMath project. Please see:

<http://3D-XplorMath.org/>