Pendulum ODE Function *

Although one may feel that the sine function is an explicit function, it is really defined via the ODE
$\sin''(x) = -\sin(x),\ \text{with}\ \sin(0) = 0, \sin'(0) = 1.$
The solutions of the ODE of the mathematical pendulum

$$f''(x) = -\sin(f(x))$$

was also studied a lot, but did not reach the popularity of the sine function. In 3D-XplorMath we use parameters as follows:

$$f''(x) = \frac{-cc}{aa} \cdot \sin(aa\ f(x)), \quad f(0) = hh, f'(0) = bb.$$

For better comparison with the sine function, $aa = 0$ is allowed. The size of cc changes the frequency, as with the sine function. In addition observe that, for $aa \neq 0.$ the period becomes longer when the amplitude grows. As with the sine function one has an "energy integral":

$$\frac{f'(x)^2}{2} - \frac{cc}{aa^2}\cos(aa \cdot f(x)) = \frac{f'(0)^2}{2} - \frac{cc}{aa^2}\cos(aa \cdot f(0)).$$

With $aa = 1, cc = -1$ we obtain solutions of the Sine-Gordon equation $q_{xt} = \sin(q)$, namely $q(x,t) := f(x+t)$. For $f(0) = \pi, f'(0) = 2$ we get $f(u) = 4\arctan(\exp(u))$.

H.K.

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* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/