

User Curves By Curvature And Torsion*

The exhibit allows to create examples for the standard Frenet theory of space curves. The initial dialogue allows to input user choices for curvature and torsion as functions of arc length, $\kappa(s), \tau(s)$.

The solution curves are programmed as if they were explicitly parametrized. Therefore all the Action Menu entries for parametrized curves are also available for these ODE-defined curves.

The differential equations in question are the famous

Frenet-Serret Equations:

$$\begin{aligned}\dot{e}_1(t) &:= \kappa(t) \cdot e_2(t), \\ \dot{e}_2(t) &:= -\kappa(t) \cdot e_1(t) - \tau(t) \cdot e_3(t), \\ \dot{e}_3(t) &:= \tau(t) \cdot e_2(t).\end{aligned}$$

For given continuous functions κ, τ these differential equations have — for given orthonormal initial values — unique orthonormal solutions $\{e_1(t), e_2(t), e_3(t)\}$.

The curve $c(t) := \int^t e_1(s) ds$ is then parametrized by arc length and has the given curvature functions κ, τ .

H.K.

* This file is from the 3D-XplorMath project. Please see:

<http://3D-XplorMath.org/>